# 24 First Order Diff Eqs

(great for differential equations students, #DiffEqMarathon)

Video: https://youtu.be/e-cTygNbEUE

**Separable**

$$\frac{dy}{dx}=g(x)h(y)$$

$$\frac{dy}{dx}=G\left(\frac{x}{y}\right)$$

sub $v=\frac{y}{x}$

$$\frac{dy}{dx}=G(ax+by+C)$$

sub $v=ax+by+c$

**Clairaut**

$$y=xy'+\varphi(y')$$

differentiate first

**Linear**

$$\frac{dy}{dx}+P(x)y=Q(x)$$

integrating factor $\mu(x)=e^{\int P(x)dx}$

**Bernoulli**

$$\frac{dy}{dx}+P(x)y=Q(x)y^r$$

sub $v=y^{1-r}$ $r$ any real

**Riccati**

$$\frac{dy}{dx}=a(x)+b(x)y+c(x)y^2$$

sub $y=y_1+v$

where $y_1$ is a particular sol

**Exact**

$$\frac{\partial F}{\partial x}dx+\frac{\partial F}{\partial y}dy=0$$

for some $F(x,y)=C$

check for exactness

$$\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y}\right)$$

**Almost Exact**

$$M\,dx+N\,dy=0$$

try $\mu(x)=e^{\int\frac{M_y-N_x}{N}dx}$

or $\mu(y)=e^{\int\frac{N_x-M_y}{M}dx}$



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(Q1.) $(\sin y - y\sin x)dx + (\cos x + x\cos y)dy = 0$

(Q2.) $(4x^2 - 10y)dx + \left(\frac{2x^3}{y} - 15x\right)dy = 0,\ y(2) = -1$ hint: S.I.F. $\mu(x,y) = x^m y^n$

(Q3.) $\frac{dy}{dx} = \cos x - y\sec x$, y(0)=2

(Q4.) $(xy + y^2 + x^2)dx - x^2 dy = 0$

(Q5.) $(2x + \cos y)dx + (x^2 \tan y + \cos^2 y)dy = 0$

(Q6.) $(x^2 + 1)\frac{dy}{dx} + xy - x = 0$

(Q7.) $\frac{dy}{dx} = 2 - \sqrt{2x - y + 3}$

(Q8.) $y' = -y(1 + xy^2),\ y(0) = \frac{1}{2}$

(Q9.) $e^y dx - e^{-x} dy = 0,\ y(0) = 0$

(Q10.) $xy' + (1 + x)y = \sin x$

(Q11.) $\frac{dy}{dx} = \frac{1 - 2y - \sin^2(x^2 y)}{x}$

(Q12.) $\frac{dy}{dx} = \sqrt{y} - y$

(Q13.) $y = xy' + \sqrt{(y')^2 + 1}$

(Q14.) $\frac{dy}{dx} = \frac{x - y}{x + y}$

(Q15.) $\frac{dy}{dx} = \frac{xy^2 - \sin x\cos x}{y(1 - x^2)},\ y(0) = 4$

(Q16.) [Logistic diff eq] $\frac{dP}{dt} = KP\left(1 - \frac{P}{M}\right)$

(Q17.) [Gompertz diff eq] $\frac{dP}{dt} = P(a - b\ln P)$

(Q18.) $(-3y + x^2y^2)\,dx + \left(x - 2x^3y + \frac{x^4}{y}\right)dy = 0$

(Q19.) $y = xy' - e^{y'}$

(Q20.) $2xye^{x^2}dx + \left(e^{x^2} - \frac{1}{y}\right)dy = 0$

(Q21.) $y' = -x + \frac{1}{2x}y,\ y(1) = 0$

(Q22.) $y' = -x + \frac{1}{2x}y + y^2,\ y(1) = 0$

(Q23.) $y' = x^2 + 2xy + y^2,\ y(1) = 0$

(Q24.) $(y^2 - x^2)dx - xydy = 0,\ y(1) = 3$

(A1.) exact, $x\sin y + y\cos x = C$

(A2.) almost exact, $\mu(x,y) = xy^2$, $x^4y^2 - 5x^2y^3 = 36$

(A3.) linear, $y = \dfrac{x - \cos x + 3}{\sec x + \tan x}$

(A4.) homogeneous, $y = x\tan\left(\ln|x| + C\right)$

(A5.) almost exact, $\mu(y) = \sec y$, $x^2 \sec y \tan y + \sin y = C$

(A6.) linear, $y = 1 + \dfrac{C}{\sqrt{1+x^2}}$

(A7.) sub $v = 2x - y + 3$ , $y = 2x + 3 - (\tfrac{1}{2}x + C)^2$

(A8.) Bernoulli, $y = \dfrac{\sqrt{2}}{\sqrt{9e^{2x} - 2x - 1}}$

(A9.) separable, $y = -\ln(2 - e^x)$

(A10.) linear, $y = \dfrac{Ce^{-x}}{x} + \dfrac{\sin x}{2x} - \dfrac{\cos x}{2x}$

(A11.) sub $v = x^2y$, $y = \dfrac{\tan^{-1}\left(\frac{1}{2}x^2 + C\right)}{x^2}$

(A12.) separable or Bernoulli, $y = \left(1 + Ce^{\frac{-1}{2}x}\right)^2$ missing solution $y = 0$

(A13.) Clairaut, general solution: $y = Cx + \sqrt{C^2 + 1}$, singular solution $y = \sqrt{1 - x^2}$

(I put $x^2 + y^2 = 1$ for singular solution in the video and that was a mistake)

(A14.) homogeneous or exact, $x^2 - 2xy - y^2 = C$

(A15.) exact, $x^2y^2 - y^2 - \sin^2 x = -16$

(A16.) separable, $P = \dfrac{M}{1 + Ce^{-kt}}$ and $C = \dfrac{M - P_0}{P_0}$

(A17.) separable $y = e^{\frac{a}{b}(1 - e^{-bt})} {p_0}^{e^{-bt}}$

(A18.) almost exact, $\mu(x) = \dfrac{1}{x^4}$ , $\dfrac{y}{x^3} - \dfrac{y^2}{x} + \ln y = C$

(A19.) Clairaut, general solution: $y = Cx - e^C$, singular solution $y = x\ln x - x$

(A20.) exact, $ye^{x^2} - \ln|y| = C$

(A21.) linear, $y = \frac{2}{3}\sqrt{x} - \frac{2}{3}x^2$

(A22.) Riccati, $y_1 = \sqrt{x}$, $y = -\sqrt{x}\,\dfrac{e^{\frac{4}{3}x^{\frac{3}{2}}} - e^{\frac{4}{3}}}{e^{\frac{4}{3}x^{\frac{3}{2}}} + e^{\frac{4}{3}}} = -\sqrt{x}\tanh\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{3}\right)$

(if would be much easier if you use $y = y_1v$ for this problem instead of $y = y_1 + v$ )

(A23.) rewrite $y' = (x+y)^2$ and sub $v = x + y$, $y = -x + \tan\left(x + \frac{\pi}{4} - 1\right)$

(A24.) homogeneous, Bernoulli, almost exact with $\mu(x) = x^{-3}$, $y = x\sqrt{9 - 2\ln x}$