Calculus 2: Exam On Series & Power Series

(Q1.) $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2} = ?$

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{2}{5}$

(D) $\frac{4}{3}$ (E) $\frac{3}{4}$

(Q2.) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

(A) converges by the Root Test (B) diverges by the Root Test

(C) converges to 1/e by "The Fact" (D) diverges by Test For Divergence

(E) I don't know, I don't care

(Q3.) Which of the following x-value **cannot** be plugged into $x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\cdots=\ln(1+x)$?

(A) 2 (B) 1 (C) 0

(D) $\frac{1}{2}$ (E) $\frac{-1}{2}$

(Q4.) Use the Taylor formula to determine the **first four nonzero terms** of the power series for

$$\sin x \text{ at } a=\frac{\pi}{4}$$

(A) $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)-\frac{1}{2!\sqrt{2}}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!\sqrt{2}}\left(x-\frac{\pi}{4}\right)^3+\cdots$

(B) $\frac{1}{\sqrt{2}}+\frac{1}{3!\sqrt{2}}\left(x-\frac{\pi}{4}\right)^3-\frac{1}{5!\sqrt{2}}\left(x-\frac{\pi}{4}\right)^5-\frac{1}{7!\sqrt{2}}\left(x-\frac{\pi}{4}\right)^7+\cdots$

(C) $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)+\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)^3+\cdots$

(D) $1-\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\cdots$

(E) $\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3-\frac{1}{5!}\left(x-\frac{\pi}{4}\right)^5-\frac{1}{7!}\left(x-\frac{\pi}{4}\right)^7+\cdots$

(Q5.) $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{3^n} = ?$

(A) $\frac{-1}{12}$ (B) $\frac{1}{12}$ (C) $\frac{3}{4}$

(D) $\frac{2}{3}$ (E) $\frac{-2}{3}$ (F) $\frac{-3}{4}$

(Q6.) Pikachu knows $R=3$ and $a=2$ for power series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2 3^n}(x-2)^n$. Help him to determine the **interval of convergence**.

(A) $I=[-1,5]$ (B) $I=[-1,5)$ (C) $I=(-1,5)$

(D) $I=(-3,3)$ (E) $I=[-3,3]$

(Q7.) Determine if $\frac{1}{3}-\frac{1}{6}+\frac{1}{11}-\frac{1}{18}+\frac{1}{27}-\cdots$ converges or not. Justify your answer.

(Q8.) Integrate the followings as a **power series**. State the **radius** of convergence

(a) $\int xe^{x^3}\,dx$

(b) $\int \tan^{-1}(x^2)\,dx$

(Q9.) Determine if $\sum_{n=1}^{\infty}\left(1-\cos\left(\frac{1}{n}\right)\right)$ converges or not. Justify your answer.

(Q10.) Give an example of…

(a) $\sum_{n=1}^{\infty}a_n$ that the Ratio Test will give inconclusive result

(b) $a_n\neq 0$ for any n but $\sum_{n=0}^{\infty}a_n=0$

(Q11.) Let $a_n=\frac{n!}{n^n}$

(a) Does a_n converge?

(b) Does $\sum_{n=1}^{\infty}a_n$ converge?

(Q12.) Determine the **power series** for $\frac{x^3}{9-x^2}$ at $a=0$ in sigma notation.

State the **radius** & **interval** of convergence

(Q13.) Determine if $\sum_{n=3}^{\infty}\frac{1}{n\sqrt{\ln n}}$ converges or not. Justify your answer.

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