100 Trinomials

Great for Algebra Students

Dedicated to Mr. Dennis Hill

Video: https://youtu.be/LmbqB_pfcDU

Every factorable "general" trinomial, $ax^2 + bx + c$, has an associated "simple" trinomial, $x^2 + bx + ac$, that is, by comparison, easy to factor, and its factors can then be used to find, by inspection, those of the general trinomial.

Example: Factor $6x^2 + 17x - 10$. Its simple version is $x^2 + 17x - 60$, which factors into $(x + 20)(x - 3)$. After making "6" the leading coefficient of each factor, getting $(6x + 20)(6x - 3)$, we then "reduce" to obtain $(3x + 10)(2x - 1)$.

In other words, when we install the leading coefficient of our general trinomial as the leading coefficient of each factor of our associated simple trinomial and then divide each of those newly-modified factors by its GCF, we obtain the factors of the original general trinomial.

Proof: Assume, for all values of x, that $ax^2 + bx + c = \underline{(dx + e)(fx + g)}$ and that $x^2 + bx + ac = (x + h)(x + k)$.

Then, after multiplying the factors of the right sides of the above equations and then equating coefficients of similar terms, we arrive at: $a = df$, $b = dg + ef = h + k$, $c = eg$, $ac = hk = (df)(eg) = (dg)(ef)$.

Since $h + k = dg + ef$ and $hk = (dg)(ef)$, then either $h = dg$ and $k = ef$, or $h = ef$ and $k = dg$, because when one pair of numbers has the same sum and product as a second pair of numbers, the two pairs must be identical. *(Proof below.)

Case 1: $h = dg$ and $k = ef$. Then $(ax + h) = (dfx + dg)$, $(ax + k) = (dfx + ef)$, $(ax + h)/d = \underline{(fx + g)}$, and $(ax + k)/f = \underline{(dx + e)}$.

Case 2: $h = ef$ and $k = dg$. Then $(ax + h)/f = \underline{(dx + e)}$ and $(ax + k)/d = \underline{(fx + g)}$.

*Proof: Let $m + n = p + q$, and let $mn = pq$. Assume $m \neq p$. Then, if $m > p$, let $m = p + j$, where $j > 0$. To maintain the given $m + n = p + q$, we must have $n = q - j$. So, $mn = (p + j)(q - j) = pq - jp + jq - j^2$. Since $mn = pq$, we have $jp = jq - j^2$ and, with j nonzero, we have $p = q - j$. So $p = n$, implying $m = q$. Therefore, if m does not equal p, then m *must* equal q, and n *must* equal p. [We get an identical argument if we start with $m < p$ and let $m = p + j$, where $j < 0$.]

Revised version. July 15th, 2019

There are 8 quadratic expressions that are NOT factorable!
(you can factor out a negative sign for one of them tho)

(Q1.) $x^2 - x - 6 =$

(Q2.) $x^2 + 2x - 15 =$

(Q3.) $3x^2 - 3x - 6 =$

(Q4.) $4x^2 + 4x - 15 =$

(Q5.) $6x^2 + 5x + 1 =$

(Q6.) $2x^2 + 6x + 4 =$

(Q7.) $6x^2 + 7x - 10 =$

(Q8.) $4x^2 - 8x + 3 =$

(Q9.) $10x^2 + 9x - 9 =$

(Q10.) $9x^2 + 10x + 9 =$

(Q11.) $15x^2 - 11x + 2 =$

(Q12.) $8x^2 - 12x - 8 =$

(Q13.) $14x^2 + 13x + 3 =$

(Q14.) $8x^2 + 14x - 15 =$

(Q15.) $2x^2 + 15x - 38 =$

(Q16.) $x^2 - 16x + 64 =$

(Q17.) $6x^2 - 17x + 12 =$

(Q18.) $3x^2 + 18x + 27 =$

(Q19.) $4x^2 + 19x - 5 =$

(Q20.) $33x^2 + 20x + 3 =$

(Q21.) $2x^2 - 21x - 50 =$

(Q22.) $8x^2 - 22x - 21 =$

(Q23.) $10x^2 + 23x - 5 =$

(Q24.) $6x^2 + 24x - 30 =$

(Q25.) $20x^2 - 25x - 30 =$

(Q26.) $5x^2 - 26x + 24 =$

(Q27.) $9x^2 + 27x - 10 =$

(Q28.) $4x^2 + 28x + 13 =$

(Q29.) $5x^2 - 29x + 36 =$

(Q30.) $7x^2 + 30x - 12 =$

(Q31.) $29x^2 + 31x + 2=$

(Q32.) $64x^2 + 32x + 4 =$

(Q33.) $24x^2 + 33x - 30 =$

(Q34.) $20x^2 + 34x + 6 =$

(Q35.) $6x^2 - 35x - 6 =$

(Q36.) $18x^2 + 36x + 10 =$

(Q37.) $12x^2 - 37x + 21 =$

(Q38.) $8x^2 - 38x + 24 =$

(Q39.) $13x^2 + 39x - 52 =$

(Q40.) $-x^2 - 40x + 50 =$

(Q41.) $-21x^2 - 41x - 10 =$

(Q42.) $-9x^2 - 42x - 49 =$

(Q43.) $-x^2 + 43x + 44 =$

(Q44.) $-x^2 - 44x - 43 =$

(Q45.) $-198x^2 - 45x + 27 =$

(Q46.) $-36x^2 + 46x + 12 =$

(Q47.) $-12x^2 + 47x - 40 =$

(Q48.) $-12x^2 + 48x + 60 =$

(Q49.) $-6x^2 - 49x - 65 =$

(Q50.) $9x^2 + 50x - 18 =$

(Q51.) $18x^2 + 51x - 42 =$

(Q52.) $72x^2 + 52x + 8 =$

(Q53.) $14x^2 + 53x + 14 =$

(Q54.) $27x^2 + 54x - 81 =$

(Q55.) $25x^2 - 55x - 12 =$

(Q56.) $32x^2 - 56x - 16 =$

(Q57.) $99x^2 + 57x - 6 =$

(Q58.) $20x^2 + 58x + 20 =$

(Q59.) $5x^2 - 59x - 12 =$

(Q60.) $5x^2 - 60x - 12 =$

(Q61.) $10x^2 - 61x - 26 =$

(Q62.) $20x^2 - 62x - 28 =$

(Q63.) $2x^2 - 63x - 32 =$

(Q64.) $16x^2 + 64x + 64 =$

(Q65.) $45x^2 + 65x - 50 =$

(Q66.) $9x^2 - 66x + 121 =$

(Q67.) $20x^2 + 67x + 56 =$

(Q68.) $24x^2 + 68x - 56 =$

(Q69.) $16x^2 + 69x + 20 =$

(Q70.) Solve $x^2 + 70x + 1200 = 0$

(Q71.) Solve $8x^2 - 71x - 9 = 0$

(Q72.) Solve $72^2 - 72x - 80 = 0$

(Q73.) Solve $x^2 - 73x - 74 = 0$

(Q74.) Solve $73x^2 - 74x + 1 = 0$

(Q75.) Solve $90x^2 - 75x - 375 = 0$

(Q76.) Solve $40x^2 + 76x + 24 = 0$

(Q77.) Solve $66x^2 + 77x - 110 = 0$

(Q78.) Solve $104x^2 + 78x - 65 = 0$

(Q79.) Solve $9x^2 - 79x - 18 = 0$

(Q80.) $37x^2 - 80x - 4 =$

(Q81.) $90x^2 + 81x - 63 =$

(Q82.) $52x^2 + 82x + 6 =$

(Q83.) $x^2 - 83x + 82 =$

(Q84.) $72x^2 + 84x - 36 =$

(Q85.) $10x^2 - 85x + 150 =$

(Q86.) $129x^2 - 86x - 43 =$

(Q87.) $18x^2 - 87x - 66 =$

(Q88.) $96x^2 + 88x - 120 =$

(Q89.) $10x^2 + 89x + 34 =$

(Q90.) $2x^2 + 90x + 90 =$

(Q91.) $78x^2 - 91x - 260 =$

(Q92.) $8x^2 - 92x - 100 =$

(Q93.) $54x^2 - 93x + 18 =$

(Q94.) $24x^2 + 94x - 8 =$

(Q95.) $114x^2 - 95x - 114 =$

(Q96.) $4x^2 - 96x - 324 =$

(Q97.) $16x^2 + 97x + 6 =$

(Q98.) $56x^2 + 98x + 35 =$

(Q99.) $18x^2 + 99x - 117 =$

(Q100.) $25x^2 + 100x + 4 =$

(Q101.) How did I come up with these problems?