100 Series

(Great for Calc 2 Students)

Video: https://youtu.be/jTuTEcwvkP4

Top Four Secret Weapons

The Fact: $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{bn}=e^{ab}$

The List: As $n\to\infty$, $\ln n \ll n^p \ll b^n \ll n! \ll n^n$, where $p>0$ and $b>1$

The Limit: $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ note: please do not say by L'Hospital's Rule

Best Friend: $\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$ where $|x|<1$

@blackpenredpen

May 4th, 2019

(Q1.) $\sum_{n=1}^{\infty}\frac{1}{n}$

(Q2.) $\sum_{n=2}^{\infty}\frac{1}{\ln n}$

(Q3.) $\sum_{n=2}^{\infty}\frac{1}{\ln(n^n)}$

(Q4.) $\sum_{n=1619}^{\infty}\frac{1}{(\ln n)^{\ln n}}$

(Q5.) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\tan^{-1} n}$

(Q6.) $\sum_{n=1}^{\infty}\frac{2^n}{3^n+n^3}$

(Q7.) $\sum_{n=1}^{\infty}\frac{3^n}{2^n+n^2}$

(Q8.) $\sum_{n=1}^{\infty}\frac{n\sin^2 n}{n^3+2}$

(Q9.) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+1}}$

(Q10.) If possible, evaluate 1/2 - 1/3 + 2/9 - 4/27 + …

(Q11.) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{n}\right)$

(Q12.) $\sum_{n=3}^{\infty}\frac{1}{n^2\ln n}$

(Q13.) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}e^{\sqrt{n}}}$

(Q14.) $\sum_{n=1}^{\infty}\frac{n^n}{3^{n^2}}$

(Q15.) $\sum_{n=1}^{\infty}\frac{n^n}{(n!)^2}$

(Q16.) $\sum_{n=1}^{\infty}n\sin(\tfrac{1}{n})$

(Q17.) $\sum_{n=1}^{\infty}\frac{1}{n+3^n}$

(Q18.) $\sum_{n=1}^{\infty}\frac{\sin(2n)}{n+3^n}$

(Q19.) $\sum_{n=1}^{\infty}\frac{(-1)^n n}{3n+1}$

(Q20.) If possible, evaluate 1/2 + 1/6 + 1/12 + 1/20 + …

(Q21.) $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

(Q22.) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

(Q23.) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(Q24.) $\sum_{n=1}^{\infty} \cos^2\left(\frac{1}{n}\right)$

(Q25.) $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\ln n}$

(Q26.) $\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$

(Q27.) $\sum_{n=1}^{\infty} \frac{1}{2^{\ln n}}$

(Q28.) $\sum_{n=1}^{\infty} \frac{1}{3^{\ln n}}$

(Q29.) $\sum_{n=1}^{\infty} \frac{3n^2+n}{\sqrt{n^5+2n+1}}$

(Q30.) If possible, evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(Q31.) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(Q32.) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

(Q33.) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n^2}}}$

(Q34.) $\sum_{n=1}^{\infty} 1$

(Q35.) $\sum_{n=1}^{\infty} \frac{n^2}{2^n+3^n}$

(Q36.) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

(Q37.) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$

(Q38.) $\sum_{n=1}^{\infty} \frac{1}{\sin^4 n}$

(Q39.) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(Q40.) If possible, evaluate $\sum_{n=1}^{\infty} \frac{1}{n^3+3n^2+2n}$

(Q41.) If $\sum_{n=1}^{\infty} (a_n)^2$ converges, then $\sum_{n=1}^{\infty} a_n$ must also converge.

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(Q43.) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ must diverge.

(Q44.) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ must converge.

(Q45.) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ must also converge.

(Q46.) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge, then $\sum_{n=1}^{\infty} (a_n+b_n)$ must also converge.

(Q47.) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge and $a_n \neq b_n$, then $\sum_{n=1}^{\infty} (a_n-b_n)$ must also diverge.

(Q48.) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge, then $\sum_{n=1}^{\infty} (a_n b_n)$ must also diverge.

(Q49.) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge, then does $\sum_{n=1}^{\infty} (a_n b_n)$ must also converge.

(Q50.) If possible, evaluate $\sum_{n=1}^{\infty} 0$

(Q51.) $\sum_{n=1}^{\infty} n\sqrt{\sin\frac{1}{n^2}}$

(Q52.) $\sum_{n=1}^{\infty} (1-\sin\frac{1}{n})$

(Q53.) $\sum_{n=1}^{\infty} (1-\cos\frac{1}{n})$

(Q54.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{2^n+1}}$

(Q55.) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$

(Q56.) $\sum_{n=1}^{\infty} \frac{n-1}{\sqrt{n^3+2n+5}}$

(Q57.) $\sum_{n=1}^{\infty} \frac{n^2 2^{n+2}}{4^n}$

(Q58.) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)$

(Q59.) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)^n$

(Q60.) If possible, evaluate $\sum_{n=1}^{\infty} a_n$, where $a_1 = 9$ and $a_n = (6-n)a_{n-1}$ for $n \geq 2$

(Q61.) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{10n}}$

(Q62.) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$

(Q63.) $\sum_{n=1}^{\infty} e^{-n} \sin n$

(Q64.) $\sum_{n=1}^{\infty} \frac{\tan \frac{1}{n}}{n^2}$

(Q65.) $\sum_{n=1}^{\infty} \frac{n^{10} 4^n}{n!}$

(Q66.) $\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$

(Q67.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n!}}$

(Q68.) $\sum_{n=2}^{\infty} \frac{1}{\ln(n!)}$

(Q69.) $\sum_{n=1}^{\infty} \frac{n(n+2)}{(2n+1)^2}$

(Q70.) If possible, evaluate $\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{\frac{1}{n+2}}\right)$

(Q71.) $\sum_{n=3}^{\infty} \frac{\ln n}{n^2}$

(Q72.) $\sum_{n=1}^{\infty} \frac{n^3}{2n^5+3n-4}$

(Q73.) $\sum_{n=1}^{\infty} \left(\frac{1-2n}{3+4n}\right)^n$

(Q74.) $\sum_{n=1}^{\infty} \frac{e^n}{2^{2n-1}}$

(Q75.) $\sum_{n=1}^{\infty} \frac{n^3 - 2n - 1}{2n^5 + 3n - 4}$

(Q76.) $\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$

(Q77.) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(Q78.) $\sum_{n=1}^{\infty} \sqrt{\cos(\frac{1}{n})}$

(Q79.) $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

(Q80.) If possible, evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(Q81.) For what values of x will the series $1^x + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(Q82.) For what values of x will the series $x^1 + x^2 + x^3 + x^4 + \cdots + x^n + \cdots$ converge?

(Q83.) For what values of x will the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$ converge?

(Q84.) For what values of x will the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converge?

(Q85.) For what values of x will the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n \cdot 3^n}$ converge?

(Q86.) For what values of x will the series $\sum_{n=1}^{\infty} n! x^n$ converge?

(Q87.) For what values of k will the series $\sum_{n=1}^{\infty} \frac{1}{x(\ln x)^k}$ converge?

(Q88.) For what values of x will the series $\sum_{n=0}^{\infty} \left(\frac{1}{1-x} \right)^n$ converge?

(Q89.) For what values of x will the series $\sum_{n=0}^{\infty} \left(\sum_{m=0}^{\infty} x^m \right)^n$ converge?

(Q90.) If possible, evaluate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(Q91.) $\sum_{n=1}^{\infty} \left(\frac{\pi}{2} - \tan^{-1} n \right)$

(Q92.) $\sum_{n=1}^{\infty} \sin^2(\frac{1}{n})$

(Q93.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} - \sqrt{n+1}}$

(Q94.) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$

(Q95.) $\sum_{n=1}^{\infty}\frac{1}{e^{\sqrt{n}}}$

(Q96.) $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{n^5-1}}$

(Q97.) $\sum_{n=1}^{\infty}\ln\left(\frac{n}{n+2}\right)$

(Q98.) $\sum_{n=1}^{\infty}\frac{1}{e^n+1}$

(Q99.) $\sum_{n=1}^{\infty}\frac{1}{\ln\left(e^n-1\right)}$

(Q100.) If possible, evaluate 1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 + …

(Q101.) $\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}$

Question for you: $\sum_{n=1}^{\infty}\frac{e^n n!}{n^n}$

Send your answer to blackpenredpen@gmail.com for a potential shout out in my future videos.