Calculus 2 Exam #3

Sequence & Series (Series Convergence Test & Power Series)

(Q1.) Consider $a_n = \frac{1}{2^n}$. Which of the following is **true**?

(A) $\lim_{n\to\infty} a_n = 1$ (B) $\lim_{n\to\infty} a_n = 2$ (C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} a_n = 1$ (E) $\sum_{n=1}^{\infty} a_n = 2$

(Q2.) Determine the 100th derivative of $\ln x$

(A) $-\frac{99!}{x^{99}}$ (B) $-\frac{99!}{x^{100}}$ (C) $-\frac{100!}{x^{100}}$

(D) $-\frac{98!}{x^{100}}$ (E) $\frac{99!}{x^{100}}$

(Q3.) $\pi - \frac{1}{3!}\pi^3 + \frac{1}{5!}\pi^5 - \frac{1}{7!}\pi^7 + \cdots = ?$

(A) 0 (B) 1 (C) $\frac{\pi}{4}$

(D) π (E) ∞

(Q4.) Which of the following series **converges by the Root Test**?

(A) $\sum_{n=1}^{\infty}\left(1+\frac{1}{2n}\right)^n$ (B) $\sum_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^n$ (C) $\sum_{n=1}^{\infty}\left(\frac{1}{2}+\frac{1}{n}\right)^n$

(D) $\sum_{n=1}^{\infty}\left(2+\frac{1}{n}\right)^n$ (E) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{2n}$

(Q5.) Given $\sum_{n=1}^{\infty} a_n$ diverges. Which of the following must be **true**?

(A) $\lim_{n\to\infty} a_n \neq 0$ (B) $\lim_{n\to\infty} a_n = 0$ (C) $\lim_{n\to\infty} a_n$ does not exisit

(D) None of the above

(Q6.) Use the Taylor formula to determine the **first four nonzero terms** of the power series for

$$\frac{1}{x} \text{ at } a = 2$$

(A) $\frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x - \frac{1}{16}x + \cdots$ (B) $\frac{1}{2} - \frac{1}{2}(x-2) + \frac{1}{2}(x-2)^2 - \frac{1}{2}(x-2)^3 + \cdots$

(C) $\frac{1}{2} + \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 + \frac{1}{16}(x-2)^3 + \cdots$ (D) $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \cdots$

(E) $\frac{1}{2} + \frac{1}{4(x-2)} + \frac{1}{8(x-2)^2} + \frac{1}{16(x-2)^3} + \cdots$

(Q7.) Evaluate the followings

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$

(b) $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\cdots$

(Q8.) Determine if $\frac{1}{\sqrt{2}}+\frac{4}{\sqrt{9}}+\frac{9}{\sqrt{28}}+\frac{16}{\sqrt{65}}+\frac{25}{\sqrt{126}}+\cdots$ converges or not? Justify your answer

(Q9.) Determine if $\sum_{n=1}^{\infty} \frac{n}{n^2+3n+2}$ converges or not? Justify your answer

(Q10.) *Give an example (with an explicit formula) of the following…*

(a) a_n so that $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges

(b) a_n so that $\sum_{n=1}^{\infty} a_n = 20$

(Q11.) Determine if $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ converges or not? Justify your answer.

(Q12.) Integrate $\int \tan^{-1}(4x^2)\,dx$ as a power series (use **sigma notation**).
State the **radius of convergence**

(Q13.) Determine the power series for $\frac{x^8}{27-x^3}$ at $a=0$ (use **sigma notation**)
State the **radius** & the **interval of convergence**.

(Q14.) Evaluate $\displaystyle\int_{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\pi^{2n+1}}^{\sum_{n=0}^{\infty} \frac{n}{n^2+3n+2}} \left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right)^{\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^{n+1}} dx$