

Calculus 2 Exam #2

Numerical Integrations, Limits, Improper Integrals, Separable Differential Equations

(Q1.) Suppose $\int_1^{\infty} f(x) dx$ diverges and $f(x) > 0$ for $x \geq 1$.

Which of the following must **also diverge**?

(A) $\int_1^{\infty} (f(x)-1) dx$

(B) $\int_1^{\infty} (f(x)+1) dx$

(C) $\int_1^{\infty} (f(x))^2 dx$

(D) $\int_1^{\infty} \sqrt{f(x)} dx$

(E) $\int_1^{\infty} \frac{f(x)}{x} dx$

(Q2.) Compute T_3 for $\int_1^4 4^{\sin x} dx$

(A) 6.524

(B) 7.112

(C) 6.912

(D) 7.312

(E) 6.112

(F) none of these

(Q3.) Which of the following is a solution to $\frac{dy}{dx} = y + \sin x$ and $y(0) = 1$

(A) $y = \sin x + \cos x$

(B) $y = \frac{3}{2}e^x - \frac{1}{2}\sin x - \frac{1}{2}\cos x$

(D) $y = e^x + \sin(2x)$

(D) $y = \frac{3}{2}e^x - \frac{1}{2}\cos x + 1$

(Q4.) Which of the following is **equal to 1**?

(A) $\lim_{x \rightarrow \infty} \frac{x+1}{\ln x}$

(B) $\lim_{x \rightarrow \infty} \frac{\ln x}{x+1}$

(C) $\lim_{x \rightarrow \infty} \frac{\ln(x+1)}{\ln x}$

(D) $\lim_{x \rightarrow \infty} \ln\left(\frac{x}{x+1}\right)$

(E) none of these

(Q5.) Compute S_4 for $\int_1^3 \frac{1}{1+\ln x} dx$

(A) 1.2652

(B) 1.2691

(C) 1.2528

(D) 1.2534

(E) 1.3333

(F) none of the above

(Q6.) The air in a room with volume $200 m^3$ contains 0.15% carbon dioxide initially. Fresher air with only 0.04% carbon dioxide flows into the room at a rate of $2 m^3/min$ and the well-mixed air flows out at the same rate. Let $A(t)$ be the amount of carbon dioxide in the room at time t . What is the value of $\lim_{t \rightarrow \infty} A(t)$?

(A) $0.08 m^3$

(B) $0.8 m^3$

(C) $8 m^3$

(D) $0.3 m^3$

(E) $0.03 m^3$

(F) none of these

(Q7.) Use the **comparison theorem** to show if $\int_1^{\infty} \frac{1}{\sqrt{x} + e^x} dx$ converges or not.

Be sure to show the 3 steps.

(Q8.) Evaluate $\int_{\sqrt[3]{2}}^{\infty} \frac{1}{x\sqrt{x^3-1}} dx$ if it converges. *DO NOT USE THE COMPARISON TEST*

(Q9.) Give **an example** of the following

(a) A type 2 improper integral that converges. Give its value.

(b) A type 2 improper integral that diverges. Explain why.

(Q10.) Determine the followings. Your answers must be in exact form.

(a) $\lim_{x \rightarrow \infty} \left(\frac{x}{x+2} \right)^x$

(b) $\lim_{x \rightarrow 1^+} (\ln(x-1) - \ln(x^2-1))$

(Q11.) Given the rate of change of y with respect to t is inversely proportional to y^2 . If $y(0)=2$ and $y(2)=10$, then what is the value of $y(4)$?

(Q12.) Given $\frac{dA}{dt} = 0.0025A - m$ and $A(0) = 300000$. Find m so that $A(180) = 0$

(Q13.) Consider the integral $\int_1^3 e^{-x^3} dx$. **How large should n be** to guarantee that the **midpoint rule** approximation to the integral is accurate to within 0.0005?

(Q14.) $\frac{d^3}{dx^3} \left(\frac{\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \cdot \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}}{\int_0^x \ln t dt} \right)$